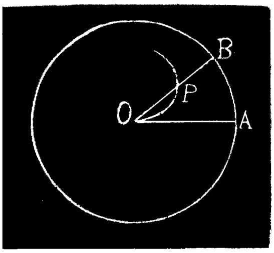
Then $us=a\theta\ldots(1)$. $\therefore\ s=\frac{a\theta}{u}=\frac{an\theta}{m}$ is the intrinsic equation to the curve.

From (1) $\frac{ds}{d\theta}=\frac{a}{u}=\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}$.

$$\therefore\ r^2+\left(\frac{dr}{d\theta}\right)^2=\frac{a^2}{u^2} \text{ and } d\theta=\frac{dr}{\sqrt{\left(\frac{a^2}{u^2}-r^2\right)}}.$$

$\therefore\ \theta=\sin^{-1}\frac{ur}{a}\ldots(2)$.



$\therefore\ r=\frac{a}{u}\sin\theta=\frac{an}{m}\sin\theta$, is the polar equation and $m(x^2+y^2)=any$, is the rectangular equation.

The value of θ from (2) in (1) gives

$$s=\frac{a}{u}\sin^{-1}\frac{ur}{a},$$

the length for any value of r. When $r=a$,

$$s=\frac{a}{u}\sin^{-1}u=\frac{an}{m}\ \sin^{-1}\frac{m}{n}=\text{distance run.}$$

$$t=\text{time}=\frac{s}{n}=\frac{a}{m}\sin^{-1}\frac{m}{n}.$$

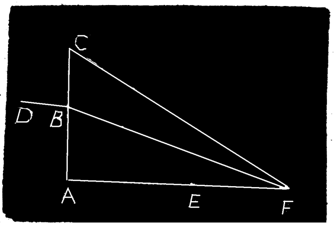
Also solved by *Professors O. W. Anthony, J. Scheffer, and William Symmonds*.

28. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

How far from the stage must Miss Love sit in order that she *may see to best advantage* Mr. Rich deliver the valedictory oration?

Solution by O. W. ANTHONY, Missouri Military Academy, Mexico, Missouri, and the PROPOSER.

Let E represent the position of Miss Love's eyes; DB the stage from which Mr. Rich orates; $AB,=m$ feet, the height of the stage above Miss Love's eyes; $BC,=n$ feet, the height of Mr. Rich; and $AE,=x$ feet, the required distance In order that Miss Love may *see* Mr. Rich *to best advantage*, the angle BEC must be a maximum, that is,



$$U=\tan^{-1}\left(\frac{m+n}{x}\right)-\tan^{-1}\left(\frac{m}{x}\right)=\text{a Maximum.}$$

$$\therefore\ \frac{dU}{dx}=\frac{m}{x^2+m^2}-\frac{m+n}{x^2+(m+n)^2}=0\ldots(1).$$

Whence $x=\sqrt{[m(m+n)]}$ feet, which is the required distance.

29. Proposed by CHARLES E. MYERS, Canton, Ohio

A hen running at the rate of $n=2$ feet per second, on the circumference of a circle, radius $r=50$ feet, is observed by a hawk $a=600$ feet directly above the center.

The hawk at once starts in pursuit, flying at the rate of $m=5$ feet per second and keeping always in a straight line with the starting point and the hen.

Determine the path followed and the distance the hawk will fly before catching the hen.

Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let the origin be at the vertex of the cone around which the path of the hawk winds. σ=length of the hawk's path, s=the length of the projection of this path on the plane (x,y), ρ=radius vector of this projection, $\frac{n}{m}=u$, $\frac{r}{a}=e$. Then $x^2+y^2=e^2z^2$ is the equation of the cone, also $n\sigma=r\theta$, where θ is the angle subtended by the hen's path at the centre of the circle.

$$\therefore \quad d\sigma=\frac{r}{u}\,d\theta=\sqrt{dx^2+dy^2+dz^2}=\sqrt{ds^2+dz^2}$$

$$=\left\{\rho^2+\left(\frac{d\rho}{d\theta}\right)^2+\left(\frac{dz}{d\theta}\right)^2\right\}^{\frac{1}{2}}d\theta,\text{ but } \rho^2=x^2+y^2=e^2z^2. \quad \therefore \quad \rho=ez, \quad \therefore \quad dz=\frac{d\rho}{e}.$$

$$\therefore \quad \frac{d\sigma}{d\theta}=\frac{r}{u}=\left\{\rho^2+\left(\frac{d\rho}{d\theta}\right)^2+\frac{1}{e^2}\left(\frac{d\rho}{d\theta}\right)^2\right\}^{\frac{1}{2}}.$$

$$\therefore \quad \frac{d\theta}{d\rho}=\frac{\sqrt{e^2+1}}{e}\cdot\frac{1}{\left\{\frac{r^2}{u^2}-\rho^2\right\}^{\frac{1}{2}}}, \quad \theta=\frac{\sqrt{e^2+1}}{e}\sin^{-1}\frac{u\rho}{r}.$$

$$\therefore \quad \theta=\frac{\sqrt{r^2+a^2}}{r}\sin^{-1}\left(\frac{n\rho}{mr}\right);\ \rho=\frac{mr}{n}\sin\frac{r\theta}{\sqrt{r^2+a^2}}.$$

$$z=\rho/e=\frac{am}{n}\sin\frac{r\theta}{\sqrt{r^2+a^2}}=1500\sin.\frac{\theta}{\sqrt{145}}.$$

$$x=\rho\cos\theta=\frac{mr}{n}\sin\frac{r\theta}{\sqrt{r^2+a^2}}\cos\theta=125\sin\frac{\theta}{\sqrt{145}}\cos\theta.$$

$$y=\rho\sin\theta=\frac{mr}{n}\sin\frac{r\theta}{\sqrt{r^2+a^2}}\sin\theta=125\sin\frac{\theta}{\sqrt{145}}\sin\theta.$$

These values of x, y, z determine the hawk's path.

$$\text{Also } n\sigma=r\theta. \quad \therefore \quad \sigma=\frac{mr}{n}\left[\frac{\sqrt{r^2+a^2}}{r}\sin^{-1}\left(\frac{n\rho}{mr}\right)\right]_0^r=\frac{m\sqrt{r^2+a^2}}{n}\sin^{-1}\left(\frac{n}{m}\right).$$

$$\therefore \quad \sigma=125\sqrt{145}\sin^{-1}\left(\frac{2}{5}\right)=619.406 \text{ feet approximately, the distance}$$

the hawk flies before catching the hen.

Solved in a similar manner by *Professor J. F. W. Scheffer.*